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RN-6165

B. E. II (Sem. III) (Comp./I.T.) Examination

May / June – 2010

Discrete Mathematics

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. 2 (Sem. 3) (Comp./I.T.)**

Name of the Subject :

☛ **Discrete Mathematics**

☛ Subject Code No. : **6 1 6 5** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) Attempt **all** questions.

(3) Figures to the **right** indicate marks.

(4) Answer **each** section **separately**.

SECTION - I

1 (a) Do as directed : **10**

(1) Define isomorphic graphs. State the necessary conditions for two graphs to be isomorphic.

(2) Define degree of vertices with illustration.

(3) Define edge-connectivity and vertex connectivity of a connected graph.

(4) Define minimally connected graph with illustration.

(5) Define binary tree.

(b) Attempt any **two** : **10**

(1) Prove that the number of vertices of odd degree in a graph is always even.

(2) Define a Hamiltonian circuit. Prove that, in a complete graph with n vertices, there are $\left(\frac{n-1}{2}\right)$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3.

(3) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

2 (a) Describe Konigsberg bridge problem. 3

(b) Attempt any **three** : 12

(1) Define centre of a tree. Prove that every tree has either one or two centres.

(2) Prove that a graph is a tree if and only if it is minimally connected.

(3) Prove that a given connected graph G is an Euler graph, if and only if all vertices of G are of even degree.

(4) Define a spanning tree. Prove that a Pendant edges in a connected graph G is contained in every spanning tree of G.

3 (a) Describe-three-uitilities problem. 3

(b) Attempt any **three** : 12

(1) Prove that a vertex V in a connected graph G is a cut-vertex if and only if there exist two vertices x and y in G, such that every path between x and y passes through V.

(2) Define a planar graph. Prove that a connected planar graph with n vertices and e edges has e–n+2 regions.

(3) Describe Kuratowski's two graphs.

(4) Define a cut-set. Prove that every circuit has an even number of edges in common with any cut-set.

SECTION - II

4 (a) Do as directed : **10**

(1) Show that $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$.

(2) Let $x = \{1,2,3,4\}$ and $R = \{(1,1),(1,4),(4,1),(4,4),(2,2),(2,3),(3,2),(3,3)\}$ write the matrix of R and sketch the graph.

(3) Given $S = \{1,2,........,10\}$ and a relation R on S where $R = \{(x,y) \mid x + y = 10\}$. What are the properties of the relation R ?

(4) Define connectives NAND and NOR.

(5) Draw Hasse diagram for (S_{30}, D) where S_{30} is the set of divisors of 30 and D stands for the relation "to be divisor of".

(b) Attempt any **two** : **8**

(1) Let $A = \{a,b,c\}$ and $\rho(A)$ its power set. Let $\subseteq$ be the set inclusion relation on the elements of $\rho(A)$. Show that $(\rho(A), \subseteq)$ is a partial ordered set.

(2) Let X=the set positive integers and D denotes relation "divides". Draw Hasse diagram for poset (S_{36}, D), where S_{36} is the set of divisors of 36. Is it a chain ? Show that (X, D) is a poset.

(3) Let I be the set of integers and $f : I \to I$ be a function defined as $f(n) = \begin{cases} n/2, & \text{if } n \text{ is even} \\ \frac{n-1}{2}, & \text{if } n \text{ is odd} \end{cases}$.

Determine whether f is one-to-one or onto or neither one-to-one nor onto.

5 (a) Attempt any **two** : 8

(1) Prove that a subset $S \neq \phi$ of G is a subgroup of $(G, *)$ if and only if for any pair of elements $a, b \in S$, $a * b^{-1} \in S$.

(2) Let $(S, *)$ be a given semigroup. Prove that there exists a homomorphism $g : S \to S^S$, where $\left(S^S, 0\right)$ is a semigroup of functions from S to S under the operation of composition.

(3) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian.

(b) Attempt any **two** : 8

(1) Show that every chain is a distributive lattice.

(2) Show that in a lattice, if $a \leq b$ and $c \leq d$, then $a * c \leq b * d$ and $a \oplus c \leq b \oplus d$.

(3) Show that in a complemented, distributive lattice, $a \leq b \Leftrightarrow a * b' = 0$.

6 (a) Attempt any **two** : 8

(1) In any Boolean Algebra, show that $a = 0 \Leftrightarrow ab' + a'b = b$.

(2) Define a complemented lattice. Show that a chain of three or more elements is not complemented.

(3) Find the complements of every element of the lattice $\langle S_{75}, D \rangle$ where D stands for the relation 'divides'. Is it a complemented lattice ?

(b) Attempt any **two** : 8

(1) Show that the function $f(x) : \left[x/2 \right]$ which is equal to the greatest integer which is $\leq x/2$, is primitive recursive.

(2) Show that the function $f(x) = \dfrac{x}{2}$ is a partial recursive function.

(3) Use the Karnaugh map representation to find a minimal sum-of-products expression for the function $f(a, b, c, d) = \sum (0, 5, 7, 8, 12, 14)$.
